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ARTICLE

L. Gmachowski

Intrinsic viscosity of bead models for macromolecules and bioparticles

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Abstract A new method based on the fractal dimension dependence of the hydrodynamic radius is proposed for calculation of the intrinsic viscosity of bead models. The method describes properly the viscosity increment except for elongated structures such as linear aggregates and ellipsoids. It is expected to be useful for very compact structures, for which the volume correction does not improve the results calculated by the modified Oseen tensor. The results obtained for the viscosity increment lie between the volume corrected ones and those determined by the cubic substitution procedure. They are close to the values recalculated from the falling velocities of the models analyzed.

Keywords Viscosity increment · Hydrodynamic radius · Fractal dimension · Sedimentation velocity · Oligomers

Introduction

The hydrodynamic properties of macromolecules and bioparticles can be represented by bead models composed of spherical elements, introduced by Bloomfield et al. (1967a, 1967b). Bead models are useful when simple geometric models, such as spheres, ellipsoids or cylinders, are inadequate owing to the specific way in which the subunits are arranged. This approach is applicable to oligomeric proteins consisting of several subunits with a polygonal or polyhedral geometry.

The method proposed by Kirkwood and Riseman (1948), based on the Oseen hydrodynamic interaction tensor between beads, makes it possible to calculate such properties as sedimentation and diffusion coefficients, as well as intrinsic viscosity. Based on that originally proposed, the method for the determination of hydrodynamic properties is improved to take into account the finite volume of beads (Carrasco and García de la Torre 1999a, 1999b).

At approximately the same time, the porous sphere model was introduced by Debye (1947) and Brinkman (1947a) and developed by Brinkman (1947b), in which a macromolecular coil was modeled as a sphere of uniform permeability. The progress of the permeable sphere model was limited, however, owing to the difficulties in connecting the permeability with the structure of the macromolecule (van Saarloos 1987).

This paper is devoted to the description of a new method for the calculation of the intrinsic viscosity using the fractal dimension dependence of the normalized hydrodynamic radius of the particle considered. This is based on an extension (Gmachowski 1996) of the Brinkman (1947c) model for fluid flow through an arrangement of permeable particles of spherical character. Therefore it is expected to be useful for compact structures. There are, however, first attempts to describe by this method the hydrodynamic behavior of linear aggregates (Gmachowski 2000) and to characterize aggregates composed of polydisperse particles (Gmachowski 1998; Bushell and Amal 2000). In perspective, this could make it possible to predict the hydrodynamic parameters of elongated structures such as linear aggregates and ellipsoids.

Model

Intrinsic viscosity is a hydrodynamic property of dissolved macromolecules or suspended particles. It is sensitive to their size and shape and easy to measure. If a solution or suspension contains particles of mass m and radius R, the intrinsic viscosity is given by the following formula:

$$[\eta] = \frac{10}{3}\pi\frac{R^3}{m}\left(\frac{r}{R}\right)^3 \tag{1}$$

where the hydrodynamic radius r is introduced of an impermeable sphere of the same mass to take into account the internal

L. Gmachowski
Institute of Physical Chemistry,
Polish Academy of Sciences, 01-224 Warsaw, Poland
E-mail: gmach@ichf.edu.pl
Fax: +48-22-6325276

permeability of real particles present. For uniformly permeable spheres of permeability k, Brinkman (1947b) calculated:

$$[\eta] = \frac{10}{3}\pi\frac{R^3}{m}\frac{1+\frac{3}{\sigma^2}-\frac{3}{\sigma\tanh\sigma}}{1+\frac{10}{\sigma^2}\left(1+\frac{3}{\sigma^2}-\frac{3}{\sigma\tanh\sigma}\right)} \tag{2}$$

where $\sigma = R/\sqrt{k}$ is the reciprocal square root of the dimensionless internal permeability of the sphere model. Hence:

$$\frac{r}{R} = \left[\frac{1+\frac{3}{\sigma^2}-\frac{3}{\sigma\tanh\sigma}}{1+\frac{10}{\sigma^2}\left(1+\frac{3}{\sigma^2}-\frac{3}{\sigma\tanh\sigma}\right)}\right]^{1/3} \tag{3}$$

For the translational friction coefficient:

$$f_T = 6\pi\eta_0 R\frac{r}{R} \tag{4}$$

Brinkman (1947b) obtained:

$$\frac{r}{R} = \frac{1-\frac{\tanh\sigma}{\sigma}}{1+\frac{3}{2\sigma^2}\left(1-\frac{\tanh\sigma}{\sigma}\right)} \tag{5}$$

It is interesting to compare both dependencies of the normalized hydrodynamic radius on the σ parameter. This is done in Fig. 1. For larger values of the σ parameter, corresponding to the values of the normalized hydrodynamic radius greater than 0.6, there is practically a unique dependence. This means that a given value of the reciprocal square root of the dimensionless internal permeability, characterizing a given particle, determines a value of the normalized hydrodynamic radius, which is the same for calculating the translational friction coefficient and the intrinsic viscosity. This also means the existence of a possibility to determine one of the two hydrodynamic parameters from the other.

To verify this possibility for oligomers, the sedimentation of an aggregate is analyzed first. The sedimentation velocity u_r of an individual aggregate of dynamic radius r composed of i sufficiently small spherical particles of radius a can be determined by equating the gravitational force, allowing for the buoyancy of the surrounding fluid, with the opposing hydrodynamic force, as given by Stokes law:

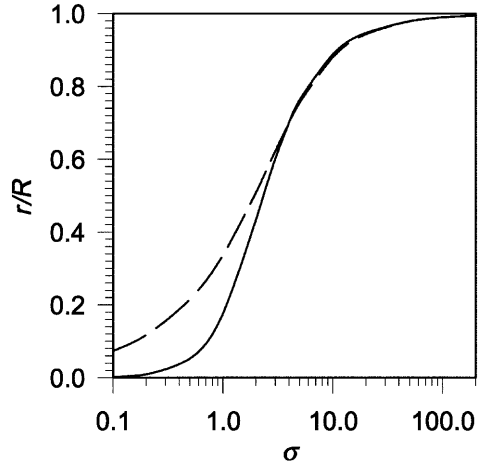

Fig. 1 Comparison of normalized hydrodynamic radii obtained from Brinkman models for intrinsic viscosity (– –) and translational friction coefficient (—) as dependent on the reciprocal square root of the dimensionless permeability of a model sphere

$$\frac{4}{3}\pi a^3(\rho_s-\rho_f)gi = 6\pi\eta_0 r u_r \tag{6}$$

with:

$$u_r = \frac{2}{9\eta_0}(\rho_s-\rho_f)g\frac{a^3 i}{r} \tag{7}$$

Using the relation between the number of constituents and the dynamic radius of a fractal aggregate (Gmachowski 2000):

$$i = \left(\frac{r}{a}\right)^D \tag{8}$$

the sedimentation velocity of an individual fractal aggregate is given as follows:

$$u_r = \frac{2}{9\eta_0}(\rho_s-\rho_f)ga^2 i^{\frac{D-1}{D}} \tag{9}$$

Relating to the Stokes velocity of a primary particle:

$$u_a = \frac{2}{9\eta_0}(\rho_s-\rho_f)ga^2 \tag{10}$$

one obtains the following equation:

$$\frac{u_r}{u_a} = i^{\frac{D-1}{D}} \tag{11}$$

from which it is possible to determine the fractal dimension of an aggregate by its mass and sedimentation velocity:

$$D = 1/\left(1-\ln\left(\frac{u_r}{u_a}\right)/\ln i\right) \tag{12}$$

The possibility to use the fractal description for aggregates composed of a relatively small number of primary particles is connected with the fact that this description is valid not only for large aggregates. This has been tested (Takayasu and Galembeck 1998) by measurements of sedimentation and diffusion coefficients for very small aggregates of i up to four. As a result, the validity of the fractal description has been confirmed down to the lower limit of particles in the cluster ($i=1$).

The intrinsic viscosity may be expressed as the product of a molecular shape parameter known as the viscosity increment v and the specific volume of the primary particles:

$$[\eta] = v\frac{(4/3)\pi a^3 i}{m} \tag{13}$$

which represents the swollen specific volume of a macromolecule because of solvent association. Comparing with Eq. (1) one obtains:

$$v = \frac{2.5}{i}\left(\frac{r}{R}\right)^3\left(\frac{R}{a}\right)^3 \tag{14}$$

from which the normalized hydrodynamic radius may be determined:

$$\frac{r}{R} = \left(\frac{vi}{2.5}\right)^{1/3}\Big/\left(\frac{R}{a}\right) \tag{15}$$

The fractal dimension dependence of the normalized hydrodynamic radius was deduced (Gmachowski 1996) based on an extension (Gmachowski 1986) of the Brinkman (1947c) model for the fluid flow through an arrangement of permeable particles. The reciprocal square root of the dimensionless internal permeability of the sphere model was found to be a function of the fractal dimension. Then it was possible to replace the real permeable aggregates by impermeable spheres of the same mass and the hydrodynamic radius r chosen such that the overall permeability remains constant. The ratio of the radii was found (Gmachowski 2000) to be the following function of the fractal dimension:

$$\frac{r}{R} = \sqrt{1.56 - \left(1.728 - \frac{D}{2}\right)^2} - 0.228 \qquad (16)$$

This relation also unifies the concentration dependence of the suspension viscosity with respect to the fractal dimension of the aggregates present, making it possible to describe finally the hydrodynamic behavior of suspensions containing aggregates by the equations of hard sphere hydrodynamics. The universal character of Eq. (16) is confirmed by the kinetics of aggregation (Gmachowski 2001).

The fractal dimension dependence of the normalized hydrodynamic radius, as derived from the Brinkman model for fluid flow through an arrangement of spherical particles, is expected to be valid for aggregates which may be treated, at least approximately, as geometrical objects of spherical character. The spherical character is lost during the growth of a linear aggregate. So the linear aggregates will not be analyzed using Eq. (16).

For several oligomeric structures the values of the normalized hydrodynamic radius were calculated by Eq. (15) from the data of the viscosity increment previously reported (García de la Torre 1989; Harding 1997; García de la Torre and Carrasco 1998), using the values of the normalized circumscribed sphere radius R/a determined for the geometries analyzed. For the same structures the values of fractal dimension were calculated by Eq. (12) on the basis of sedimentation data (Stöber and Flachsbart 1969; Stöber et al. 1969) of $(u_r/u_a)^{1/2}$ equal to 1.18, 1.34, 1.47 and 1.68 for dimer, triangular trimer, tetrahedron tetramer and trigonal prism hexamer, respectively. In Fig. 2 the values of the normalized hydrodynamic radius are depicted versus those of the fractal dimension. The points obtained are close to the solid line representing the dependence described by Eq. (16), although the values of the normalized hydrodynamic radius for more compact aggregates (tetrahedron tetramer and trigonal prism hexamer) seem to be too high. Nevertheless, the curve of the fractal dimension dependence of the normalized hydrodynamic radius corresponds well to both the values of the viscosity increment and these of the sedimentation velocity.

Results and discussion

The values of the viscosity increment for some oligomeric structures were calculated by Eq. (14), using the values of the normalized circumscribed sphere radius R/a determined for the geometries analyzed. The values of the normalized hydrodynamic radius r/R were determined by Eq. (16) together with the following form (Gmachowski 2000) of the mass-radius relation:

$$i = \left(\frac{r}{R}\right)^D \left(\frac{R}{a}\right)^D \qquad (17)$$

The results are presented in Table 1. They are compared in Fig. 3 (●) with these obtained using a modified Oseen tensor and volume correction (○) as reported by García de la Torre (1989), Harding (1997) and García de la Torre and Carrasco (1998). There also are depicted the values (△) recalculated from sedimentation velocities (Stöber and Flachsbart 1969; Stöber et al. 1969). The required values of the normalized hydrodynamic radius r/R were determined in that case by the formula derived from Eqs. (7) and (10) as follows:

Table 1 Viscosity increments calculated for different oligomeric structures by the proposed method

Oligomeric structure	Solid volume fraction in model sphere	Viscosity increment
Hexagon hexamer	0.2222	5.795
Dimer	0.2500	3.852
Pentagon pentamer	0.2537	5.203
Bipyramid pentamer	0.2739	5.016
Square tetramer	0.2843	4.475
Octahedron hexamer	0.2942	5.025
Triangle trimer	0.3000	4.227
Tetrahedron tetramer	0.3633	4.189
Trigonal prism hexamer	0.3716	4.449
Cube octamer	0.3923	4.488

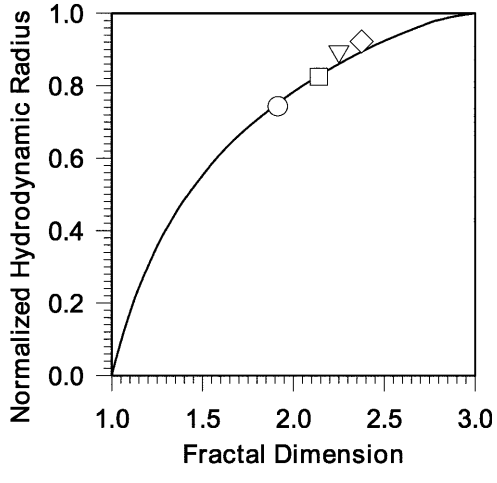

Fig. 2 Normalized hydrodynamic radius calculated by Eq. (15) versus fractal dimension determined by Eq. (12) depicted for the following models: ○ dimer; □ triangular trimer; ▽ tetrahedron tetramer; ◇ trigonal prism hexamer. The *solid line* represents the dependence described by Eq. (16)

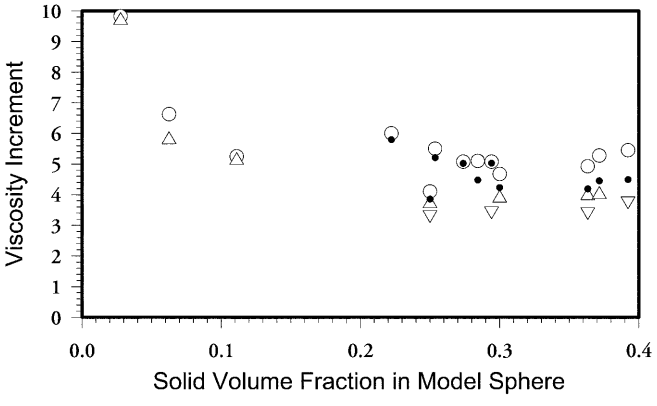

Fig. 3 Values of the viscosity increment calculated for different bead models by the following methods: ● using the proposed method; ○ using the modified Oseen tensor and volume correction as reported by García de la Torre (1989), Harding (1997) and García de la Torre and Carrasco (1998); △ from sedimentation velocities (Stöber and Flachsbart 1969; Stöber et al. 1969); ▽ using the cubic substitution method (García de la Torre and Carrasco 1998). The following oligomeric structures are analyzed (*from left to right*): linear hexamer, linear tetramer, linear trimer, hexagon hexamer, dimer, pentagon pentamer, bipyramid pentamer, square tetramer, octahedron hexamer, triangle trimer, tetrahedron tetramer, trigonal prism hexamer and cube octamer

$$\frac{r}{a} = \frac{i}{\frac{u_r}{u_a}} \Rightarrow \frac{r}{R} = \frac{i}{\frac{u_r}{u_a}\frac{R}{a}} \tag{18}$$

Finally, the values of the viscosity increment are taken into account (∇), obtained using the cubic substitution method (García de la Torre and Carrasco 1998). This method gives almost the same results as the shell model, in which the oligomer interior is inaccessible to solvent (Carrasco and García de la Torre 1999a, 1999b).

For loose structures, being linear aggregates, the values calculated by the Oseen tensor with the volume correction are close to those calculated from sedimentation data. For more compact structures the values determined by the proposed method are slightly lower than calculated by the Oseen tensor with the volume correction and slightly higher than those obtained from sedimentation data. For the most compact structures considered (tetrahedron tetramer, trigonal prism hexamer and cube octamer), the values calculated by the Oseen tensor with the volume correction seem to be too high compared to those obtained by the proposed method and from sedimentation velocities. The overestimation of the viscosity increment calculated for such compact structures by the Oseen tensor with the volume correction was reported by García de la Torre and Carrasco (1998). The results obtained by the cubic substitution method are the lowest among those analyzed. They were considered as the reference for the other procedures (García de la Torre and Carrasco 1998).

The placement of the values of the viscosity increment, calculated by the proposed method based on the fractal dimension dependence of the hydrodynamic radius, among the others available in the literature, suggests the reliability of the new method. The values calculated can be used to identify (Carrasco et al. 1998) the conformation of macromolecules and bioparticles with a greater confidence. Compared to others, the method is very simple and it is not restricted to relatively loose structures. The base of the calculations, the fractal dimension dependence of the normalized hydrodynamic radius of the model sphere, is valid up to the limit $D \rightarrow 3$, making it possible to describe the most compact structures of any mass.

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